GAT Increase Calculation for Pedestrians

Rick Bradford, 8/9/23

Contents

1. Purpose

To carry out an independent calculation of the effect on Global Average Temperature (GAT) of an increase of CO_2 concentration from the pre-industrial 280 vppm to 420 vppm (as it is in 2023, an increase of 50%).

Note that by GAT we understand the near-surface air temperature.

2. Limitations

(a) I am interested here only in the **direct** effect of CO_2, not on consequent (or feedback) effects, such as might be mediated by an associated increase in water vapour concentration, or by changes in albedo due to ice cap melting or increased cloud cover.

(b) I am interested here in radiant energy transfer and shall attempt to avoid, if possible, having to quantify other modes of energy transfer, although this will inevitably introduce approximations.

(c) I am interested only in the effect of CO_2 on the GAT. To quantify this it is necessary to deploy the idealisation that the Earth's atmosphere can be considered as a steady state with the GAT being actually manifest as a constant, uniform condition. This is a fiction. Temperatures vary markedly with location, and also vary markedly with time at every location, with both diurnal and annual periods. It is worth noting that the amplitudes of these spatial and temporal variations are far larger than the changes in GAT caused by CO_2 increases. Consequently, it is a moot question whether it is valid to introduce an approximation which is larger in magnitude than the effect being calculated. I have made no attempt to justify this.

3. Quantifying Radiant Energy Transfer

"Spectral Radiance", $I_\nu(\bar{r}, \hat{n})$, is the key quantity. It is defined as the power radiated through a small (vector) element of area $\delta\bar{A}$ into a small solid angle $\delta\Omega$ oriented about unit vector $\hat{n}$ due to radiation in the frequency range ν to $\nu + \delta\nu$. If δE_ν is the associated energy in time interval δt, then the associated flux is,

$$\delta Flux_\nu = \frac{\delta E_\nu}{\delta A \delta t} = I_\nu(\bar{r}, \hat{n})cos\theta \cdot \delta\Omega\delta\nu \tag{1}$$

where θ is the angle between $\delta\bar{A}$ and $\hat{n}$.
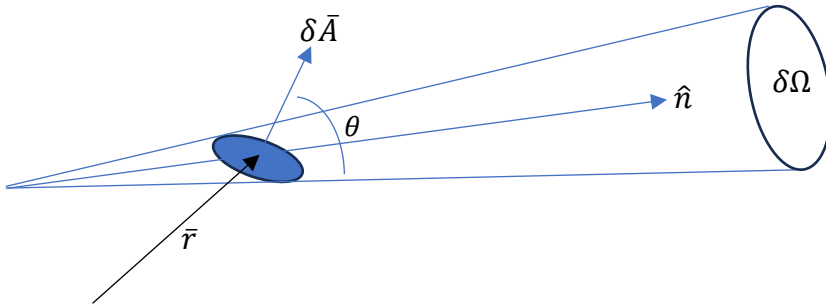
The units of spectral radiance are thus $Wm^{-2}Hz^{-1}steradian^{-1}$. However, authors commonly use wavenumber as a substitute for frequency, where wavenumber, k, is defined as the reciprocal of wavelength, $k = \frac{1}{\lambda} = \frac{\nu}{c}$. Moreover, it is usual to express wavenumber in

cm^{-1}, so if we take c as defined by 2.998×10^8, i.e., in m/s, then the spectral radiance in units $Wm^{-2}steradian^{-1}$ per cm^{-1} is $\widetilde{I}_k(\bar{r}, \hat{n}) = 100cI_v(\bar{r}, \hat{n})$ so that,

$$\widetilde{I}_k(\bar{r}, \hat{n})cos\theta \cdot \delta\Omega\delta k = I_v(\bar{r}, \hat{n})cos\theta \cdot \delta\Omega\delta v \tag{1a}$$

4. Isotropic Radiation

If the radiation is isotropic, i.e., $I_v(\bar{r}, \hat{n})$ is independent of $\hat{n}$, then the net energy through the element of area is zero, because the energy in one direction cancels with that in the reverse direction. Algebraically, $\int_{all\ directions} cos\theta \cdot d\Omega = 0$.

However, if we are interested in the energy flowing in one direction through the element of area only (as we will be) then we need to integrate over only 2π steradians, and we have,

$$\int_{forward\ direction} cos\theta \cdot d\Omega = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} cos\theta. d(cos\theta)\, d\varphi = 2\pi.\frac{c^2}{2}\Big|_0^1 = \pi \tag{2}$$

Hence, if we want the total energy passing from underneath to above a horizontal element of area, due to isotropic radiation, then,

Total flux upwards: $\qquad\qquad \delta Flux_v = \pi I_v(\bar{r})\delta v \tag{3}$

Blackbody radiation is isotropic, hence the importance of these observations.

5. Absorption

"Absorption" of radiation is just that: at the photon level, the photon is absorbed by an atom or molecule by raising its energy state. This may mean raising the energy state of an atomic orbital electron or a molecular orbital electron (more likely for optical or ultraviolet frequencies), or raising the vibrational or rotational energy state of the whole molecule (more likely for infrared frequencies).

For a given sample of gas, or other medium, the "absorption coefficient", α, is defined as the fractional reduction of radiance per unit distance travelled through the medium,

$$\frac{\delta I_v}{I_v} = -\alpha\delta x \tag{4}$$

Hence, the absorption coefficient has dimensions of 1/length. In obvious notation, integration gives, assuming α is constant,

Absorption Only, Uniform Conditions $\qquad I_v(x) = I_v(0)e^{-\alpha x} \tag{5}$

Clearly, the absorption coefficient defined in this way will depend upon the number of molecules available to do the absorbing per unit distance traversed through the material. That is, the greater the number density of molecules, ρ_N, the greater the absorption. We can therefore define a "molecular absorption coefficient", α_{mol}, by factoring out the number density dependence, $\alpha_{mol} = \alpha/\rho_N$, giving a measure of the absorption per molecule. The result has the dimensions of area and is the same thing as the absorption cross-section, σ_a.

Absorption Cross-Section $\qquad\qquad \sigma_a = \alpha_{mol} = \alpha/\rho_N \tag{6}$

Note that if we were interested only in the absorption due to one particular species, e.g., CO2, then ρ_N would be the number density of CO2 molecules only. By dividing (6) by the molecular mass, M, we get the Mass Absorption Coefficient: $\alpha_{mass} = \alpha/M\rho_N$, where $M\rho_N$ is the partial density of the species in question.

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Note that it is the cross-section, or equivalently the mass absorption coefficient, which is the fundamental quantity, independent of density, whereas the absorption coefficient, α, will depend upon the density of the medium. Here we shall use, equivalently, the mass absorption coefficient (σ_a/M) as the fundamental quantity. It has units m^2/kg.

If there are several species of absorbing molecules present, then the total reduction in radiance over distance δx will be $\delta I_{v1} + \delta I_{v2} + \delta I_{v3} + \cdots$, where δI_{v1} is the reduction in radiance due to molecular species 1, etc. Each one of these terms is given by an equation like (4), so that the total reduction in radiance is given by,

$$\frac{\delta I_v}{I_v} = -\delta x \sum_i \alpha_i = -\delta x \sum_i \rho_i \alpha_{mass,i} \tag{7}$$

where the subscript i refers to the molecular species, and ρ_i is its partial density, $\rho_i = M_i \rho_{Ni}$. The total absorption coefficient that enters the attenuation equation, (5), is thus,

$$\alpha = \sum_i \alpha_i = \sum_i \rho_i \alpha_{mass,i} \tag{8}$$

The presence of partial density in (8) shows how the absorption coefficient becomes far smaller at higher altitudes where the atmospheric density becomes very tenuous.

Note that α and $\alpha_{mass,i}$ will be (strongly!) frequency dependent, though that dependence has not been explicitly displayed above.

6. Scattering, Extinction, Opacity

These terms will not be needed but the following points are noted for clarity (as they can cause confusion when reading other sources).

In scattering, the radiation (or photon) is not absorbed but deflected into a different direction of travel. In general this will also involve a loss of energy, and hence the radiation (photon) emerges with reduced frequency. One can define a scattering coefficient in a manner analogous to the absorption coefficient.

Extinction is the term given to the attenuation of radiance due to the combination of both absorption and scattering. Thus, the extinction coefficient is the sum of the absorption coefficient and the scattering coefficient.

Similarly, the mass scattering coefficient and mass extinction coefficient are defined by factoring by the partial density, as for absorption.

"Opacity", a term used in astrophysics, is another name for the mass extinction coefficient.

7. Mean Free Path and Optical Depth

Given the attenuation equation, (5), what is the average distance travelled by a photon before it is absorbed (i.e., the mean free path, Λ)? Equ.(5) says that the probability that a photon will travel a distance x and still not be absorbed is $e^{-\alpha x}$. Hence, the probability that absorption will occur between x and $x + \delta x$ is $-d(e^{-\alpha x}) = \alpha e^{-\alpha x} dx$. The mean distance before absorption is thus,

$$\Lambda = \int_0^\infty x \, \alpha e^{-\alpha x} dx = -\alpha \frac{\partial}{\partial \alpha} \int_0^\infty e^{-\alpha x} dx = -\alpha \frac{\partial}{\partial \alpha} \frac{e^{-\alpha x}}{-\alpha}\Big|_0^\infty = -\alpha \frac{\partial}{\partial \alpha} \left(\frac{1}{\alpha}\right) = \frac{1}{\alpha} \tag{9}$$

i.e., the mean free path is the reciprocal of the absorption coefficient. If there are several absorbing species, then the mean free path is the reciprocal of the total absorption coefficient.

Another way of expressing this is that the mean free paths of the species taken separately combine in the manner analogous to resistors in parallel, i.e., $\frac{1}{\Lambda} = \frac{1}{\Lambda_1} + \frac{1}{\Lambda_2} + \cdots$.

If the composition and density of a gas were constant over some region, then passage through a distance L would be L/Λ mean free paths. A distance through the medium normalised in this way is called the "optical depth". More generally, if composition or density varies along the path length then the optical depth between positions x_1 and x_2 is given by,

Optical Depth: $\qquad\qquad\qquad \tau(x_1, x_2) = \int_{x_1}^{x_2} \frac{dx}{\Lambda} = \int_{x_1}^{x_2} \alpha dx$ $\qquad\qquad$ (10)

8. Absorption Through Non-Uniform Media

If α varies through the medium, then integration of (4) involves precisely the integration of (10), and we have…

Absorption Only, Non-Uniform Conditions $\quad I_\nu(x_2) = I_\nu(x_1)e^{-\tau(x_1,x_2)}$ $\qquad\qquad$ (11)

9. Emission and Blackbody Spectral Radiance

A body, or surface, at a non-zero absolute temperature inevitably emits radiation.

Blackbody radiation can be defined as the radiation that occurs in equilibrium inside a closed cavity whose walls are at a uniform absolute temperature T. The spectral radiance of blackbody radiation is given by the famous Planck formula,

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/k_B T}-1}$$ $\qquad\qquad$ (12)

It is isotropic and homogeneous, so the $\bar{r}$ and $\hat{n}$ dependence of the general spectral radiance, $I_\nu(\bar{r}, \hat{n})$, of equ.(1) disappears. Here Planck's constant is $h = 6.622 \times 10^{-34} Js$ and Boltzmann's constant is $k_B = 1.38 \times 10^{-23} J/K$, and $c = 2.998 \times 10^8 m/s$. Like any spectral radiance it has dimensions such that the units are $Wm^{-2}Hz^{-1}steradian^{-1}$ (which is strictly the same as Jm^{-2}).

Following equ.(3), the total blackbody flux of energy in a given frequency interval in one sense through a surface is,

Total blackbody energy in one sense: $\qquad \delta Flux_\nu = \pi B_\nu(T)\delta\nu$ $\qquad\qquad$ (13)

Equs.(3) and (13), with their factor of π, are important because we shall concentrate upon total resolved flows of energy upwards.

As in equ.(1a), if we choose to express the blackbody spectral radiance per unit wavenumber, with wavenumber in cm^{-1}, then we get,

$$\widetilde{B_k}(T) = \frac{200h\nu^3}{c} \cdot \frac{1}{e^{h\nu/k_B T}-1}$$ $\qquad\qquad$ (14)

if we continue to set c numerically to 2.998×10^8 and want the units of $\widetilde{B_k}$ to be $Wm^{-2}steradian^{-1}\ per\ cm^{-1}$. I labour this point because it can be a source of numerical confusion.

[Aside: The spectral radiance of blackbody radiation, (11), should not be confused with its energy density per frequency interval, $u_\nu(T)$, which is related to the spectral radiance by

$u_v(T) = \frac{4\pi}{c} B_v(T)$ and has units $Jm^{-3}Hz^{-1} = Jsm^{-3}$. Again I labour the point because these pesky factors of 4π and c can be confusing].

10. Absorptivity, Emissivity and Kirchoff's Law of Radiation

A blackbody is a body or surface which absorbs all radiation incident upon it. An example is a small hole in a large cavity, because any radiation entering the hole is incident upon the internal surface of the cavity many times, after previous reflections. Whatever the fraction of radiation absorbed at each interaction, after a sufficient number of reflections all the radiation will be absorbed.

Real bodies, e.g., small volumes of a gas, will not be such perfect absorbers. For a given frequency of radiation, the fraction of the incident energy absorbed by a body is called its "spectral absorptivity", $\tilde{\alpha}(v, T)$, a pure number between 0 and 1. A blackbody is a perfect absorber and therefore has $\tilde{\alpha}(v, T) = 1$.

Similarly, for a given frequency, the energy flux emitted by a body or surface expressed as a fraction of that of blackbody radiance is called its "spectral emissivity", $\varepsilon(v, T)$.

Consider a body emersed in blackbody radiation at a given temperature, and assume equilibrium prevails (i.e., the temperature of the body and the radiation are constant, and hence the frequency spectrum of the radiation is constant). Any such body , and its surrounding radiation field, will be in equilibrium only if the body is emitting the same radiant energy at every frequency as it is absorbing. In other words, its spectral emissivity and its spectral absorptivity must be equal,

$$\tilde{\alpha}(v, T) = \varepsilon(v, T) \tag{15}$$

This is Kirchoff's Law of Radiation. Note that it holds only when comparing spectral absorptivity and spectral emissivity at the same frequency and temperature, and only under equilibrium conditions.

[Another way of arguing that $\tilde{\alpha}(v, T) = \varepsilon(v, T)$ is to appeal to time-reversal symmetry, in which case absorption becomes emission and vice-versa].

11. Absorptivity, Emissivity and the Absorption Coefficient

We now have yet another term which quantifies absorption, namely absorptivity, to add to the absorption coefficient, the mass absorption coefficient and the absorption cross-section. Their relationship is simple. Consider a small thickness of gas, Δ_x, through which the radiation travels. Equ.(5) tells us it is attenuated by the factor $e^{-\alpha\Delta_x} \approx 1 - \alpha\Delta_x$. In other words the fraction of the radiation absorbed is $\alpha\Delta_x$, i.e., displaying the frequency and temperature dependence explicitly now, the dimensionless absorptivity is given in terms of the absorption coefficient by $\tilde{\alpha}(v, T) = \alpha(v, T)\Delta_x$. This applies only if the region of thickness Δ_x has uniform conditions of composition, temperature, density, etc. Note that the absorption coefficient (dimensions 1/length) can be temperature dependent. For example, at fixed pressure the density will vary with temperature and we have seen that α is proportional to (partial) density.

Coupling this with Kirchoff's Law we therefore also know the emissivity if we know the absorption coefficient, $\tilde{\alpha}(v, T) = \varepsilon(v, T) = \alpha(v, T)\Delta_x$. This is extremely important because

of the dominance of emission in the calculation we need to carry out to achieve the stated purpose.

12. A Fallacy

Thermal (infrared) radiation emitted by the ground propagates through the atmosphere, being attenuated (partially absorbed) *en route* until the remainder escapes into space. It is seriously erroneous to think that this is a sufficient picture of how much radiation is radiated into space by the Earth. The reason: it omits emission of radiation by the atmosphere itself, at all altitudes. The latter is, in fact, the dominant issue. Only a small proportion (about 9%) of the radiation which eventually emerges into space originates from the ground (as we will see). The rest is re-radiated by emission from the atmosphere. And note that such emission is possible because the temperature of the atmosphere at each altitude is maintained by absorption. Hence, radiation makes its way through the atmosphere by being periodically absorbed and then re-radiated (in general at a different frequency). This process of multiple absorptions and re-emissions constitutes a throttling of the flow of energy. In other words, the effect of "greenhouse gasses" is to increase the impedance to the energy flow. And whilst this does indeed occur via their ability to absorb radiation, to omit the re-emission part is to make a serious blunder – in particular because increased absorption increases temperature and hence also increases emissions.

In short, it's complicated, init?

13. Taking Emission into Account

Equ.(11) tells us how the spectral radiance is attenuated due to its passage through an absorbing atmosphere, but it ignores the emission of radiation that will be occurring at every layer in the atmosphere. Here we derive how equ.(11) must be modified to include emissions.

We now introduce the assumptions of isotropic radiation and Local Thermal Equilibrium. The latter is key to quantifying emissions without the need for additional empirical input due to Kirchoff's Law giving us the emissivity as a free gift if we know the mass absorption coefficient and the density (and hence the absorption coefficient and hence the absorptivity per unit thickness).

Consider altitude x. The spectral radiance at a given frequency due to emission from the atmosphere at a lower altitude, $x' < x$, arises from the blackbody spectral radiance at the temperature prevailing at altitude x', i.e., $T(x')$, hence $B_v(T(x'))$, times the emissivity of the atmosphere at altitude x'. The latter is $\varepsilon(v, T(x')) = \alpha(v, T(x'))dx'$, where dx' is the (small) thickness from which emission is considered to arise. But $B_v(T(x'))\alpha(v, T(x'))dx'$ is only the emission which would be observed immediately above altitude x'. Between this altitude and the higher altitude, x, absorption of this radiation will occur such that the fraction of it remaining is $e^{-\tau(x',x)}$ (from equ.11). Hence, the spectral radiance at altitude x due to emission from a thickness of atmosphere dx' at altitude x' is,

$$B_v(T(x'))\alpha(v, T(x'))e^{-\tau(x',x)}dx' \qquad (16)$$

The total <u>upward</u> energy flux at altitude x due to emissions is the integral of (15) over all lower altitudes, $x' < x$. Combining this with the radiation from the ground which survives to this altitude, from (11), we have the overall governing equation for the total resolved upward energy flux at frequency v at altitude x,

$$\pi I_v(x) = \pi I_v(0)e^{-\tau(0,x)} + \pi \int_0^x B_v(T(x'))\alpha(v, T(x'))e^{-\tau(x',x)}dx' \qquad (17)$$

where I have multiplied all terms by π in accord with equ.(3) so that this is the total resolved upward flux (as opposed to radiation that happens to be travelling precisely vertically).

In deploying (16) the absorption coefficient will be replaced in terms of the sum over the products of the mass absorption coefficients of the absorbing species and their partial densities at the altitude in question,

$$\alpha(v, T(x')) = \sum_i \rho_i(x')\alpha_{mass,i}(v) \qquad (18)$$

The partial densities will depend upon the pressure and temperature at altitude x', as well as on the composition. In (18) I have introduced the approximation that the mass absorption coefficients are temperature independent. This is not exactly so, but good enough for our purposes.

The proportion of the atmosphere which is water vapour varies markedly with altitude, being far greater in the lower troposphere than higher up (due to evaporation from ground level and condensation into clouds higher up). The variation of H_2O concentration with altitude will be addressed below.

The concentration of CO_2 also varies with altitude, though far less markedly than H_2O – details below.

14. Atmospheric Strata

The atmosphere is generally sub-divide into four distinct regions according to altitude.

- the Troposphere - altitudes 0 to 11 km
- the Stratosphere – altitudes 11 to to 51 km
- the Mesosphere - altitudes 51 to 71 km
- the Ionosphere (or Thermosphere) – altitudes above 71 km

However, the boundaries are very variable, especially with latitude. Thus, the troposphere may extent to heights between 6 km (polar regions) to 18 km (tropical regions). Similarly the mesosphere may extend to 80 km or 90 km.

The regions are separated by the tropopause, the stratopause and the mesopause respectively, these being regions where the gradient of temperature is zero, temperature being a local minimum, maximum and minimum respectively (see Figure 1).

15. Concentration of H2O versus Altitude

The sources listed in the References provide the following information,

(a) Whilst the total H_2O in the atmosphere varies, Ref.[1] puts the total H_2O mass at 0.25% of the total mass of the atmosphere.

(b) Over 99% of the water vapour (99.13% according to Wiki, Ref.[3]) occurs in the troposphere, Ref.[2].

(c) Ref.[4] indicates that H_2O concentration drops to about 7 ppmv in the tropopause (altitude 11 to 15 km) and drops further to between 2 and 6 ppmv in the lower stratosphere.

(d) Ref.[5] indicates that H2O concentration at the top of the mesosphere (83 km) ranges between 1 and 6 ppmv depending on time of year, with an average around 3 ppmv.

These conditions are reasonably well reproduced by assuming the following H2O concentrations versus altitude, with linear interpolation between,

Table 1:

Altitude (km)	H2O concentration, f_{H2O}
0	0.5% by mass
13	7 ppmv × 0.621 by mass[*]
83	3 ppmv × 0.621 by mass[*]
100	1 ppmv × 0.621 by mass[*]

[*] *this factor is the ratio of H2O molecular weight to the mean molecular weight for air, i.e., 18/28.965*

Note that the partial density for H2O that occurs in (18) is then given by $\rho_{H2O} = f_{H2O}\rho$ where ρ is the total density of the atmosphere. Both terms f_{H2O} and ρ are sensitive to altitude.

16. Concentration of CO2 versus Altitude

I take the near-ground CO2 concentration in 2023 to be 420 ppmv (and 280 ppmv pre-industrial). Variations with altitude are as follows,

(a) Ref.[6] indicates that CO2 drops off by about 10 ppmv over the first 2 km of altitude, but thereafter remains constant over the troposphere.

(b) Ref.[7] gives the CO2 concentration at altitude 100 km in 2012. Accounting for the rate of increase that indicates a level of 250 ppmv in 2023/4.

(c) A rather old paper, Ref.[8], indicates a reduction in CO2 by 7 ppmv between the tropopause (say, 11km) and mid-Stratosphere (35 km).

Guided by this I have used the following CO2 concentrations versus altitude, with linear interpolations between,

Table 2:

Altitude (km)	CO2 concentration, f_{CO2}	
	2023	Pre-industrial
0	420 ppmv × 1.519 by mass[*]	280 ppmv × 1.519 by mass[*]
2	410 ppmv × 1.519 by mass[*]	270 ppmv × 1.519 by mass[*]
11	410 ppmv × 1.519 by mass[*]	270 ppmv × 1.519 by mass[*]
35	403 ppmv × 1.519 by mass[*]	263 ppmv × 1.519 by mass[*]
100	250 ppmv × 1.519 by mass[*]	167[#] ppmv × 1.519 by mass[*]

[*] *the factor is the ratio of CO2 molecular weight to the mean molecular weight for air, i.e., 44/28.965*
[#] *obtained as pro-rata with 2023, i.e., (250/420)*280*

17. Insolation at Outer Atmosphere

The temperature of the "surface" of the Sun is an ill-defined concept. The photosphere is at about 5500K, but the corona is far hotter, exceeding a million K. The effective temperature is defined as that which will correctly reproduce the total radiated power per unit area via Stefan's Law, $\sigma_S T^4$ where $\sigma_S = 5.67 \times 10^{-8} W/m^2 K$. The Sun's effective temperature is 5775K, giving the energy flux at its surface to be $6.307 \times 10^7 W/m^2$.

This is then subject to reduction according to the inverse square law over the distance between the Sun and the Earth. The area of the sun is $4\pi R^2$ where $R = 6.96 \times 10^8 m$ is the Sun's radius. The energy it radiates is spread uniformly over a sphere of area $4\pi D^2$, where $D = 1.496 \times 10^{11} m$ is the Sun-Earth distance (averaged over the Earth's elliptical orbit). Hence the energy flux at the Earth is a fraction $(R/D)^2 = 2.165 \times 10^{-5}$ times $6.307 \times 10^7 W/m^2$ giving 1.365 kW/m². The resolved area that the Earth presents perpendicularly to the incident radiation is πr^2 so this is the area by which 1.365 kW/m² must be multiplied to get the total power incident on the Earth. But as we are interested in global averages this power must be averaged over the whole surface of the Earth (as the temporal averaging means the whole surface is exposed at some times), which is $4\pi r^2$. Hence, the global-temporal average insolation at the outer atmosphere is only one-quarter of 1.365 kW/m², hence 341.3 W/m².

This incident radiation has a spectrum which is, of course, maximum in the optical range.

[An exercise for the reader is to derive Stefan's constant $\sigma_S = \frac{2\pi^5 k^4}{15 c^2 h^3}$ by integration of (12), and using (13), over all frequencies].

18. Albedo and a Key Calculational Assumption

The average albedo of the Earth, i.e., the proportion of the incident radiant energy which is reflected back into space, is ~0.30. About 78% of the reflected radiation is reflected off clouds, the rest is reflected from the ground (including the oceans and ice caps).

Hence the net power (of incoming, mostly optical radiation) which is absorbed by either the atmosphere or the ground is 0.70 of 341.3 W/m², hence **238.9 W/m²**.

For the Earth to be in equilibrium, this is the total power which must be re-radiated back into space by emission from the ground plus emission from the atmosphere, and this will be almost entirely in infrared frequencies (due to the temperature of the ground and the atmosphere). Consequently, the figure 238.9 W/m² will play a crucial role in my calculations as it will be the total power of the out-going infrared radiation which my estimates must reproduce.

The important point is that this must be the total out-going power both before and after increases in CO2, because the figure is constrained to equal the net incoming power – and that depends only on the incident power and the albedo which is not *directly* related to CO2. Of course, feedback effects may very well cause changes to the albedo (e.g., greater cloud cover, reduced ice-cap cover, etc) but the remit of my calculation is the *direct* effect of CO2, ignoring feedback effects.

19. The Earth's Effective Temperature

Equating $\sigma_S T^4$ to 238.9 W/m² gives $T = 254.8K$ (-18.4ºC).

It is often said that this would be the GAT if there were no "greenhouse gases". However, as 33% of the net downward energy flux remaining after reflection (238.9 W/m²) is currently absorbed by the atmosphere (though in the visible or near-visible frequency range) it is not clear to me that this claim is valid. If the atmosphere were completely transparent to radiation of all frequencies then more down-coming radiation would reach the ground and hence the ground would reflect more radiation. The result would be that, overall, the Earth would

absorb less than 238.9 W/m^2, and hence would reradiate less in the infrared wavelengths. This suggests a GAT even lower than -18.4°C.

20. A Standard Atmosphere

Recall that we will be employing the fiction that the atmosphere is static with global and temporal average properties. Rather than calculate from first principles the atmosphere's temperature, pressure and density as a function of altitude I shall assume one of the many available "standard atmospheres", specifically the US Standard Atmosphere 1976, obtained from Ref.[9].

However, it will be necessary to change the temperatures of the standard atmosphere, both due to global warming and in order to tune the calculated infrared emission (see Solution Procedure).

In particular I have tweaked the pre-industrial GAT at ground level to be 15.85°C (289.0K) so that the resulting radiation flux ($\sigma_S T^4$) aligns with accepted values, namely 395.5 W/m^2.

21. Tweaking the Standard Atmosphere

Suppose we need to consider a change, $\Delta T(x)$, to the standard temperature distribution, $T(x)$, so that the new temperature distribution is $T'(x) = T(x) + \Delta T(x)$. Given any such temperature distribution the pressure and density can be found from the gas law and the equation of mechanical equilibrium, i.e.,

$$P = \rho_m RT \quad \text{and} \quad P = \int_0^x g\rho dx \tag{19}$$

where ρ_m is the molar density of the air, $R = 8.317\ JK^{-1}mole^{-1}$ is the gas constant, and P is the pressure (in Pa when density is in kg/m^3). The acceleration due to gravity, g, has been written inside the integral because it varies with altitude. I have used $g = 9.80665\ m/s^2$ at ground level. Where variation with altitude was included this uses the factor $\left(1 - \frac{2x}{R_{Earth}}\right)$.

Equs.(19) were solved by numerical integration using step sizes of 1 metre.

Figure 1: Standard temperature profile v altitude (after minor tweaking)

temperature (K) v altitude (m) (2023/Zhong/TD5)
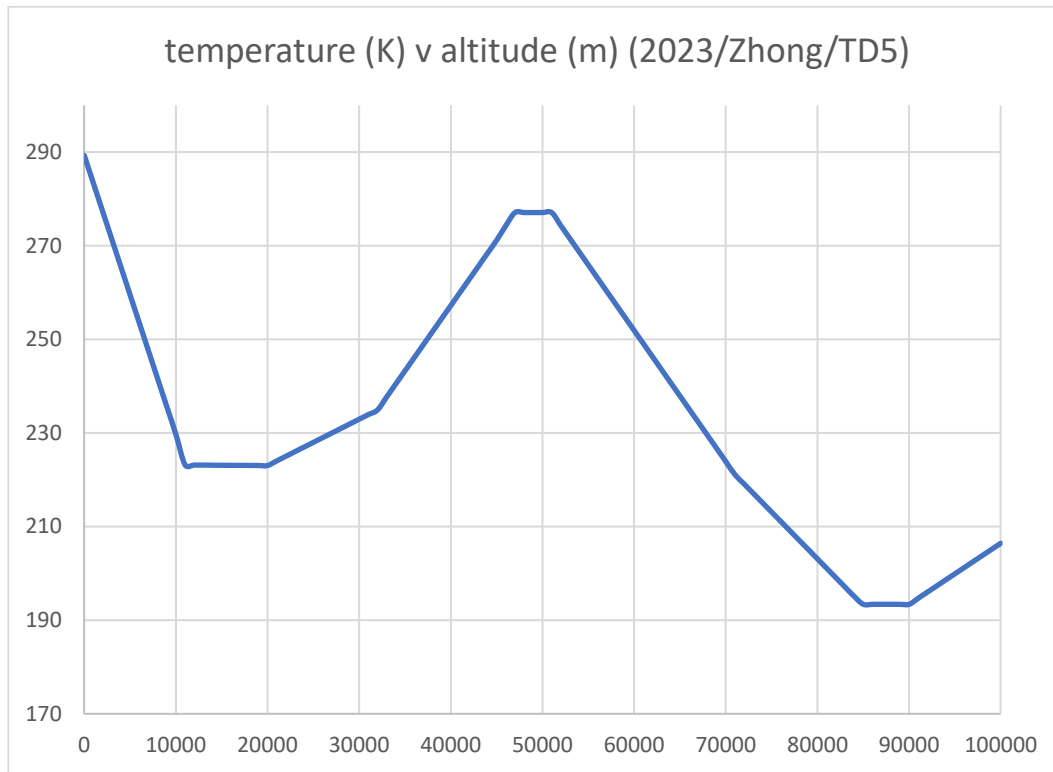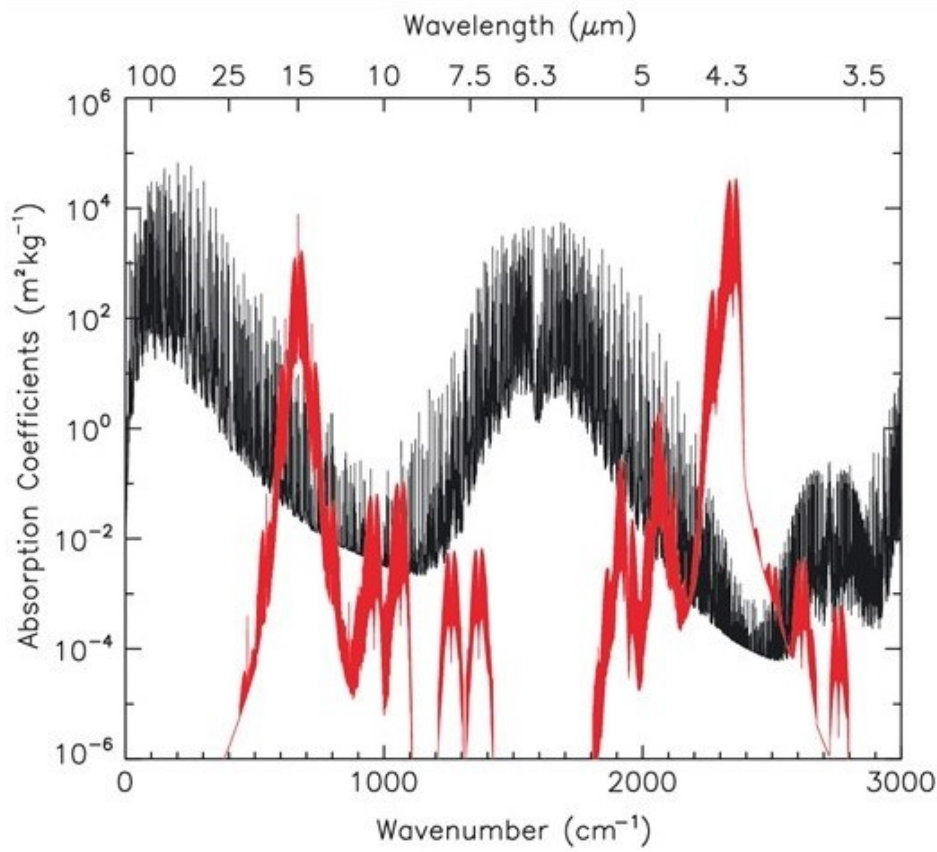
22. The H2O Mass Absorption Coefficient Used

Two different sources of mass absorption coefficient have been used. The first was from Ref.[10], reproduced as Figure 2 (units m2/kg). The second source was Ref.[11], which gives the absorption coefficient (units m^{-1}), as shown as Figure 3 (green line). This was converted to mass absorption coefficient by dividing by the density of water, 1000 kg/m^3. I have deployed these by taking a grid of 36 to 50 points and linearly interpolating between them. There is a broad band of values displayed in Figures 2 and 3, so for this purpose I have merely picked a central value by eye.

When these mass absorption coefficients are tweaked to reproduce the correct emission into space, pre-industrial, and without any simultaneous tweaks to the temperature profile, the results are shown in Figure 3. Note that these are the $\alpha_{mass,H2O}$ per wavenumber interval, dk, with k in cm^{-1}.
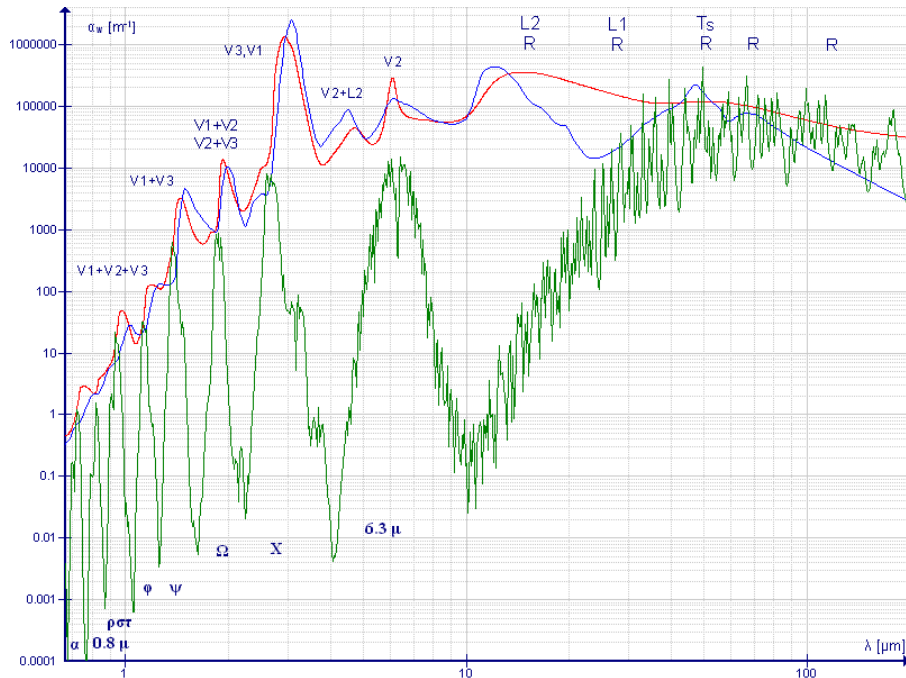
23. The CO2 Mass Absorption Coefficient Used

This was also taken from Ref.[10]. As shown in Figure 1. Picking a number of discrete points and using linear interpolation between them, the resulting formulation actually used here is shown in Figure 4. This is the $\alpha_{mass,CO2}$ per wavenumber interval, dk, with k in cm^{-1}.
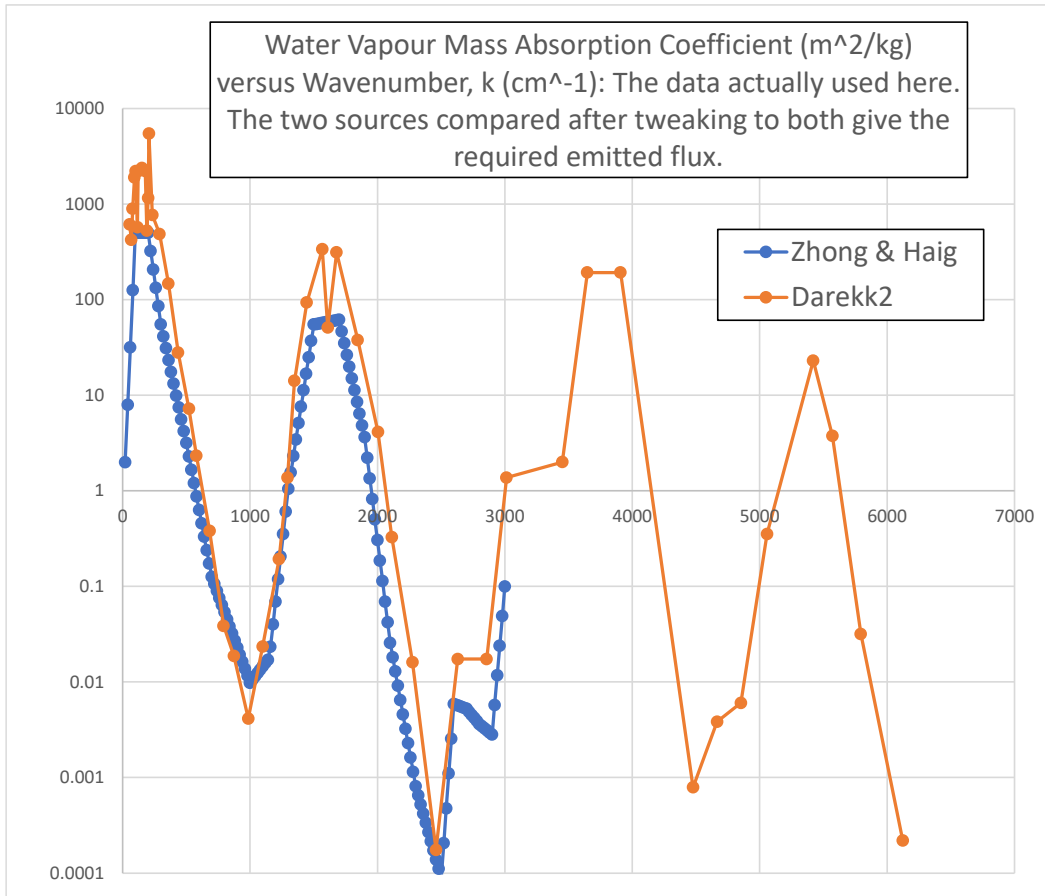
Figure 2: H2O and CO2 Mass Absorption Coefficients from Ref.[10] (Zhong)

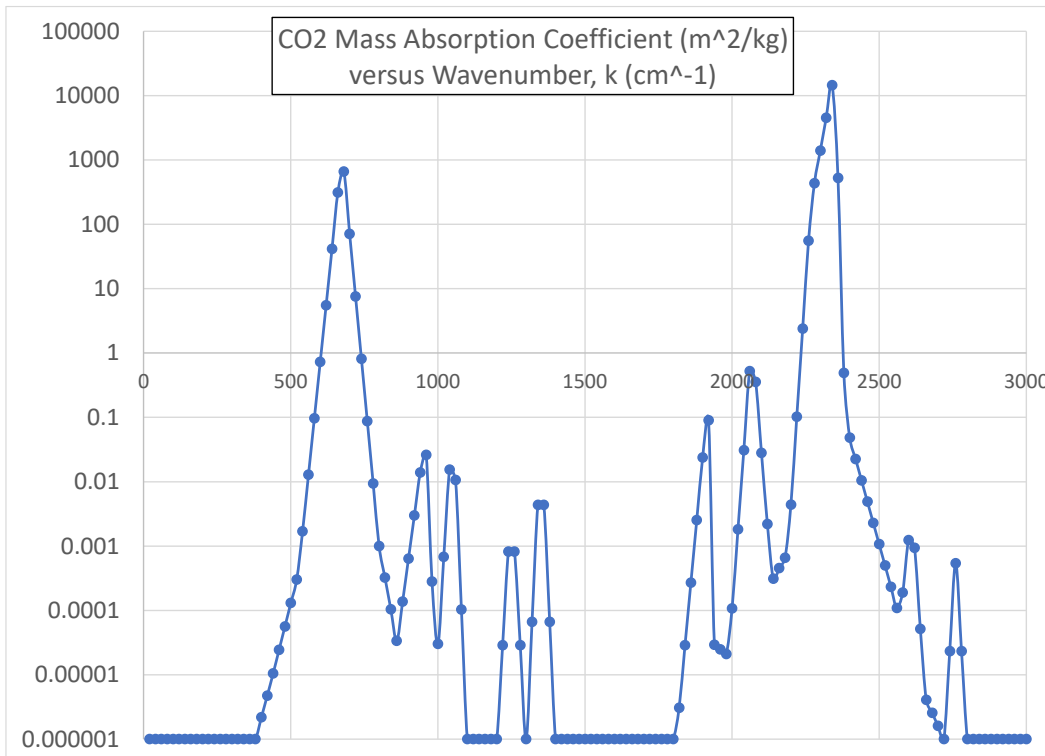

Figure 3: H2O absorption coefficient v wavelength (green = vapour), from Ref.[11] (Darekk2)

Figure 4: H2O mass absorption coefficient used in calculations after tweaking to reproduce the required emission into space (pre-industrial)

Figure 5: The mass absorption coefficient of CO2 as actually used

24. Solution Procedure

I am attempting an estimate of the effect on GAT of an increase in CO2 concentration.

I am NOT attempting a "dead reckoning" calculation of the atmosphere's altitude distribution of temperature, pressure and density from first principles…only the perturbation of these from a standard condition due to an increase in CO2. The latter is a small (very small) perturbation.

What I have already presented is,

[1] The assumed "standard atmosphere";

[2] Data for the H2O and CO2 absorption coefficients in the infrared regime;

[3] Data on H2O and CO2 concentration as a function of altitude;

The solution procedure will be,

[4] Tweak [1] and/or [2] so as to reproduce the known emitted flux of 238.9 W/m^2 at the top of the atmosphere;

[5] Introduce the increase in CO2, from 280 vppm to 420 vppm (a factor of 1.5 on CO2 absorption) to see how much the emitted flux is reduced thereby - other things (temperatures, densities) being unchanged;

[6] Assume how the change of temperature due to CO2 varies with altitude (see §25, below).

[7] Re-calculate the densities and pressures consistent with the changed temperature profile;

[8] Run the model with both the CO2 increase and the changed temperature profile and consistent densities.

[9] Repeat [6], [7], [8] with different amplitudes of temperature change until the required emission of 238.9 W/m^2 is re-established. This provides the final result, i.e., the temperature change at ground level consistent with the increased CO2.

Explanatory Notes

- The emission flux of 238.9 W/m^2 is invariant and functions as the target for the above procedure because it is a fixed requirement to balance the incoming absorbed energy flux, the latter being defined by the insolation at the outer atmosphere minus the energy reflected, neither of which is ***directly*** affected by CO2.

- The tweaking referred to in [4] consists either of making changes to the temperature profile or making changes to the mass absorption coefficient of H2O. The H2O mass absorption coefficient is tweaked by applying a factor which is the same at all wavelengths. The temperature tweak is taken to be zero at the ground and to increase linearly up to height 10,000 metres, and thereafter is constant. By keeping the near-ground temperature unchanged the flux of upward infrared radiation from the ground is invariant at 395.5 W/m^2.

  Note that these tweaks are applied to the pre-industrial conditions in order to reproduce the emission into space of 238.9 W/m^2. These tweaks are then left unchanged when the CO2 concentration is increased to model the 2023 condition.

- Obtaining a reasonable altitude profile for the temperature change is crucial. The stratosphere and mesosphere actually reduce in temperature due to increased CO2, and this affects the ground level temperature increase substantially. Assuming a uniform temperature increase at all altitudes under-estimates the ground-level temperature increase, as we will see.

Approximations

- I have taken the ground temperature to be the same as the ground-level air temperature (recalling that we are dealing with spatial and temporal averages), so that the upward incident radiation at the ground corresponds to temperature 15.85°C, i.e., 395.5 W/m^2.

- I have ignored the actual spherical geometry and treated the problem as strictly 1D. Since I consider altitudes up to 100,000 m (1.57% of the Earth's radius) the effect might be significant.

- Initially I ignored the variation of g with altitude, but I shall include cases which take the reduction of g into account (-3.14% at 100,000 m). This will reduce pressures and hence densities and hence reduce absorption.

25. Models for the Temperature Change Profile

These are displayed by Figure 6. The whole profile is scaled by the temperature change at ground level, ΔT, such that the required the emission into space of 238.9 W/m^2 is regained.

The most onerous profile, leading to the largest ΔT, is TD3. This may also be the most likely. It has long been a prediction of models that CO2 increases lead to a temperature *reduction* in the stratosphere. Recent measurements, Ref.[12], have confirmed this, and indicate that this temperature reduction is roughly double the temperature increase at ground level. Ref.[12] gives temperature changes since 1985 against altitude as,

*Table 3: Measured temperature changes since 1985(Ref.[12])*

Altitude, km	Temperature change, °C
3.1	0.821
5.6	0.705
19	-0.564
30	-1.75
38	-1.95
45	-2.22

Hence the motivation for my model TD3.

That my calculations here do not provide a means of determining the variation of the temperature change with height is their chief shortcoming. Unfortunately, the larger the temperature reduction in the stratosphere/mesosphere, the larger the temperature increase in the troposphere needs to be to reproduce the required emission at the top of the atmosphere.

Figure 6:

Temperature Profile Models, TD1 to TD5
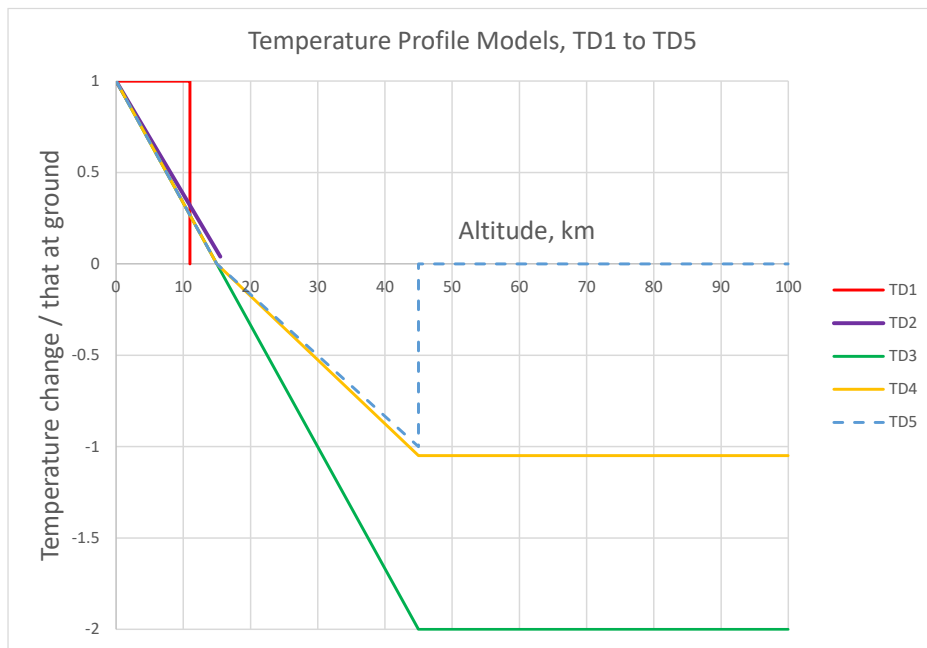
26. Results

The results are presented in Table 4. The first column indicates if the H2O mass absorption coefficient has been tweaked to get the pre-industrial emission correct, or the temperatures. The second column indicates which of the five temperature profiles of Figure 6 were used. The Table gives the predicted temperature increase at ground level, i.e., the GAT increase. The first figure assumes g is constant, whilst the figure in brackets is for the altitude-dependent g.

How do my 'predictions' compare with the IPCC models? To-date, empirical data suggests a GAT increase of around 1.1ºC to 1.2ºC. As I discuss in Ref.[13], however, only just under half of this (0.57ºC) is predicted to arise from the direct effects of CO2 radiative forcing. This figure of 0.57ºC is therefore to be compared with my Table 4 results. The average of all my results in Table 4 is 0.54ºC. I'm not sure this averaging makes much sense, but I tentatively conclude that the IPCC predictions of the pure CO2 forcing effect, without feedbacks, is reasonable, i.e., close to my average value.

Actually, I suspect that tweaking the H2O mass absorption coefficient is the better approach, in which case my results range from 0.29ºC to 0.73ºC.

However, the IPCC models actually include assumed feedback effects (mostly due to water vapour and hence cloud cover) but these are introduced as tuneable parameters which are fitted to the historic data, rather than true predictions. By means of these fitted parameters agreement with the empirical data, giving a total GAT increases to-date of 1.1ºC to 1.2ºC results – but only because it has been fitted. More of that below.

Table 4: Results for ground-level temperature (GAT) increase by 2023 (variable g in brackets)

Tweak	Model	Darekk2	Zhong
H2O	TD1	0.29 (0.31)	0.34 (0.35)
H2O	TD2	0.34	0.42 (0.44)
H2O	TD3	0.70 (0.73)	0.68
H2O	TD4	0.45	0.52
H2O	TD5	0.40	0.47
Offset	TD1	0.77 (0.77)	0.20
Offset	TD2	0.87	0.26
Offset	TD3	1.08 (1.09)	0.54
Offset	TD4	0.97	0.35
Offset	TD5	0.92	0.30

The delicacy of this calculation is illustrated by how little the radiant flux into space is affected by the 50% increase in CO2 (before temperature change is included), as given I Table 5.

Table 5: Percentage change in emission from 50% increase in CO2 (variable g in brackets)

Tweak	Darekk2	Zhong
H2O	0.26% (0.28%)	0.35%
Offset	1.01% (1.02%)	0.19%

To emphasise the dominance of re-radiation by the atmosphere, the proportion of the infrared emission into space which originates directly from the ground (i.e., the first term in equation 17) is in a single figure of percent. Ref.[10] implies about 9%, but my calculations give not more than 2.2%.

27. Putting the Results in Context

Table 5 – the percentage change in emissions due to CO2 increase - is, perhaps, a more important result than the increase in GAT (Table 4). The reduction in radiant energy emissions due to the 50% increase in CO2 (were temperatures not to increase to compensate) are only 0.2% to 1%. In absolute terms this is a decrease in infrared emissions into space of around 0.5 to 2.4 W/m^2.

How does this small change compare with other variabilities in the climate?

By using the GAT we have obviated discussion of the diurnal and seasonal variations in temperature, as well as the variations with latitude. These three variations might be roughly about 6%, 13% and 23% of the absolute temperature - vastly greater than the CO2 induced changes in infrared emission. This calls into question whether working on the basis of a fictional uniform, average temperature, with static unvarying temperatures, may be misleading – or, at least, might induce an error which swamps the apparent signal.

Of even greater concern (to me, anyway) is cloud cover. Typically 67% of the Earth is covered by cloud. But this varies from day to day by around 30% (i.e., 52% to 82%). Averaging over a year causes the variation in annual averages to be much smaller, of course, but it is still around 3% (e.g., 66% - 69%). In absolute numbers this implies a variability in

the (short wavelength) energy reflected off clouds of 3% of 79 W/m^2, i.e., 2.4 W/m^2. This is greater than, or at best equal to, the change in infrared emissions due to a 50% CO2 increase. This does not mean that natural variations in cloud cover would prevent a GAT increase of ~1ºC from being observable (clearly, it *is* observable), but it does mean that it could not be discerned from a single year. Rather, many years of data are required so that the random natural variations average to smaller percentages.

However, what it does illustrate is that an increase in ***average*** cloud cover comparable to its annual variability (over the time required to increase CO2 by around 50%, say some tens of years) would be sufficient to negate the effects of that CO2 increase on GAT completely. In other words: just how confident can we possibly be that cloud cover increases would not negate GAT increases due to CO2 completely, even if the latter were to double?

As noted above, the IPCC models include feedback effects, largely as fitted parameters. To fit the past data, cloud effects are required to be ***positive*** feedback effects, see Ref.[13]. Why should we regard a positive feedback due to cloud cover to be credible? This is not clear to me as increasing temperatures lead to increasing water vapour concentrations which one might expect, perhaps naively, to increase cloud cover rather than to decrease it – and hence to be a negative feedback.

The other variability which has received some attention is the natural variability of the power of the Sun. Readers will be familiar with the 11 year period solar sunspot cycle, the Schwabe cycle. Solar radiance is related to sunspot activity (the more sunspots, the greater the annual average solar power). Over the 11 year cycle, the minimum-to-maximum range in radiance is about 0.1%. However, on very short periods of 10 days during the solar maximum, dips in radiance of ~0.3% can occur due to occlusion by large groups of sunspots.

Popular-level discussions of the impact of solar variation on the Earth's climate often address only the 11 year Schwabe cycle, and note that a 0.1% variation in the Sun's output would cause about a 0.1ºC to 0.2ºC variation in the GAT. However, this would be oscillatory, not cumulative, and so could not account for the observed 1.1ºC increase in GAT since "pre-industrial". An example of this kind of discussion is Ref.[14], which opines that "*solar influence is not expected to dominate climate change*". Despite that, the same reference notes that changes in solar activity might have been responsible for well-known historical changes in climate, thus,

"*Almost no sunspots were observed on the sun's surface during the period from 1650 to 1715. This extended absence of solar activity may have been partly responsible for the Little Ice Age in Europe and may reflect cyclic or irregular changes in the sun's output over hundreds of years. During this period, winters in Europe were longer and colder by about 1 C than they are today. Since then, there seems to have been on average a slow increase in solar activity.*"

The solar activity, and hence the Sun's radiance, is, in fact, subject to a number of other cycles of widely varying period. Strictly these are not regular cycles, but have rather irregular periods. The next in terms of period length is the Gleissberg cycle, which many sources claim to have a period of 88 years, although the latest cycle was over 100 years. Ref.[15] isolates the Gleissberg cycle clearly by working with a measure of how much the ***cumulative*** number of sunspots differs from its mean, see Figure 7. This cumulative count can extend over several centuries. By this means a far wider minimum-maximum range in solar activity can

be identified (with a period of about a century) than occurs over the 11-year Schwabe cycles (the small ripples in Figure 7 are the Schwabe cycles). Figure 4 suggests a range of about 0.4%.

The obvious concern with using a measure based on cumulative solar activity is whether this can be expected to relate to GAT changes. The radiance of the Sun depends upon its current activity, not upon some measure of a running average activity. The latter gives only an idea of the integrated energy over large (century-wide) periods of time. Would that lead to cumulative effects on the GAT? I don't know. And is the relatively large thermal inertia of the oceans relevant in making such long timescale integrated effects relevant? I don't know, perhaps.

Figure 7: The Gleissberg cycle (from Ref.[15])
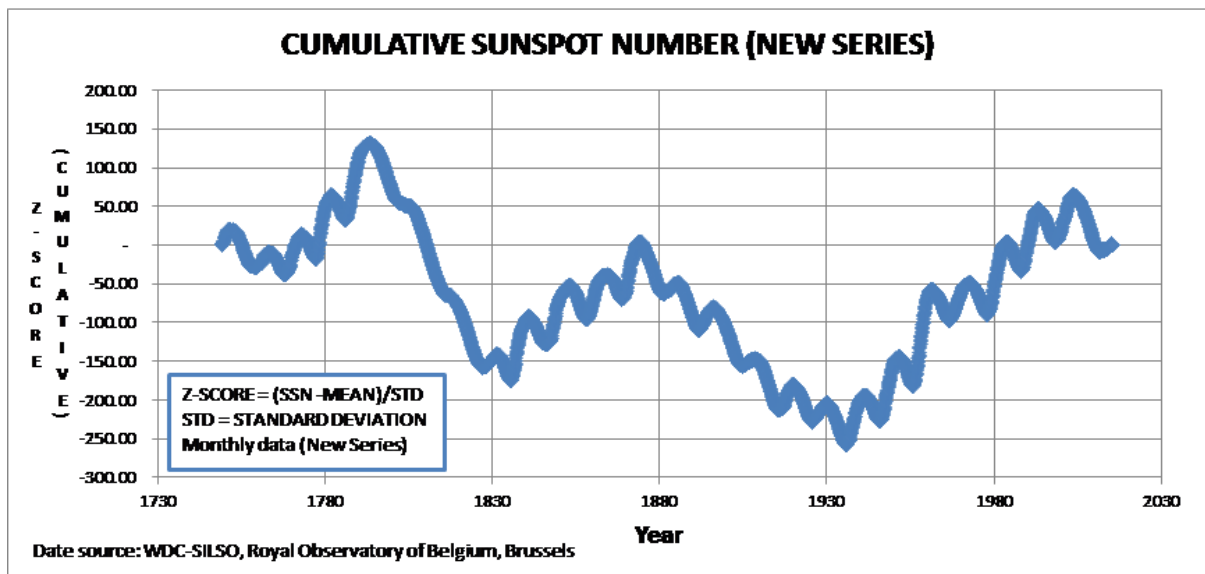
However, there is other evidence of unusually increased solar activity over the last couple of centuries derived from carbon-14 data, e.g., Refs.[16,17], illustrated by Figure 8. Based on these and similar publications, Wiki notes that,

"*The level of solar activity beginning in the 1940s is exceptional – the last period of similar magnitude occurred around 9,000 years ago (during the warm Boreal period). The Sun was at a similarly high level of magnetic activity for only ~10% of the past 11,400 years.*"

Figure 8: Solar activity as measured by carbon-14
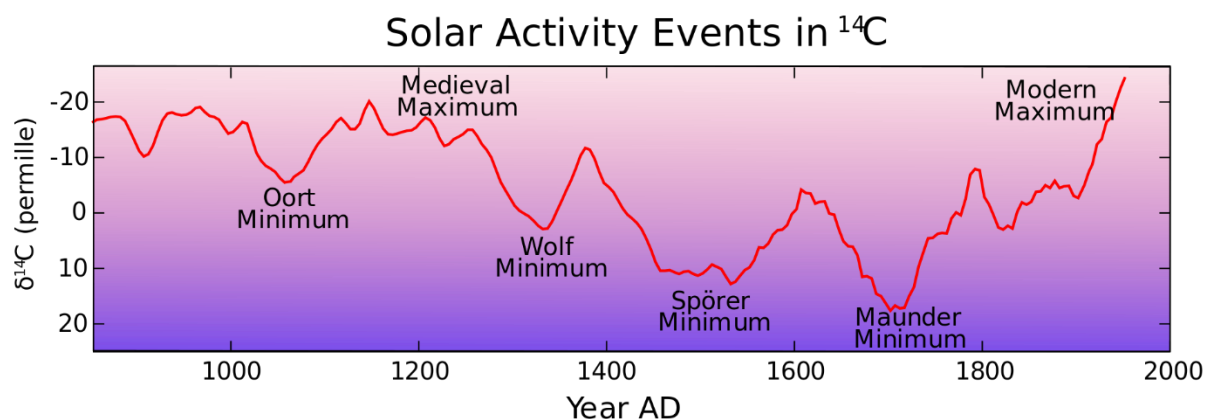
The next recognised solar cycle in terms of period is the de Vries cycle, with a cycle of around 200 years, though variable. This is the cycle which is claimed to be responsible for the obvious minima shown in Figure 8: the Maunder minimum in ~1700, the Sporer minimum in ~1500, the Wolf minimum in ~1300 and the Oort minimum a bit before 1100. A number of researchers claim to have seen evidence for variations in the abundance of certain cosmogenic isotopes such C^{14} and Be^{10}, for example Ref.[18]. The solar activity minima are relate to peaks in C^{14} abundance, measured from tree rings in the case of Ref.[18].

Then there are the effects on insolation of the Earth's orbital changes. Although of debatable relevance to variations in GAT over a couple of centuries, it is of some interest to note how large are the variations in insolation resulting from long timescale variations in the Earth's orbit. The three parameters which characterise the Earth's orbit are its eccentricity, its obliquity and its precession. The most important of these as regards variability is the obliquity. (Obliquity is the angle between the equatorial plane and the plane of the ecliptic). Ref.[19] gives a nice description of these effects, from which Figure 9 was taken.

The timescale of these variations is tens of thousands of years – hence my scepticism of their relevance. Nevertheless, it is of interest to note how comparatively huge are the magnitudes of these variations. Since the Younger Dryas cool period some 12 thousand years ago, the increase in insolation due to the Earth's orbital changes has been more than 5%. Over the period covered by Figure 9 the minimum-maximum variation has been a massive 20% Compare this with the Table 5 changes due to a 50% increase in CO2.

The significance of these orbital change issues to the contemporary global warming issue comes when models of GAT changes are deployed over the last 10,000 years. Ref.[19] shows that falling obliquity results in a good reproduction of a falling GAT over this period, and that this stands in contrast to CO2/methane based models which incorrectly predict a rising GAT over the period (see Figure 10).

Figure 9: The variability of the Earth's orbital parameters (from Ref.[19]). The last glacial period is shaded in blue and the present day is shown with the heavy vertical line. For reference the last glacial maximum (LGM) and the Younger Dryas cool period (YD) are marked. The bottom graph is the computed mean daily insolation at 65°N on the summer solstice.
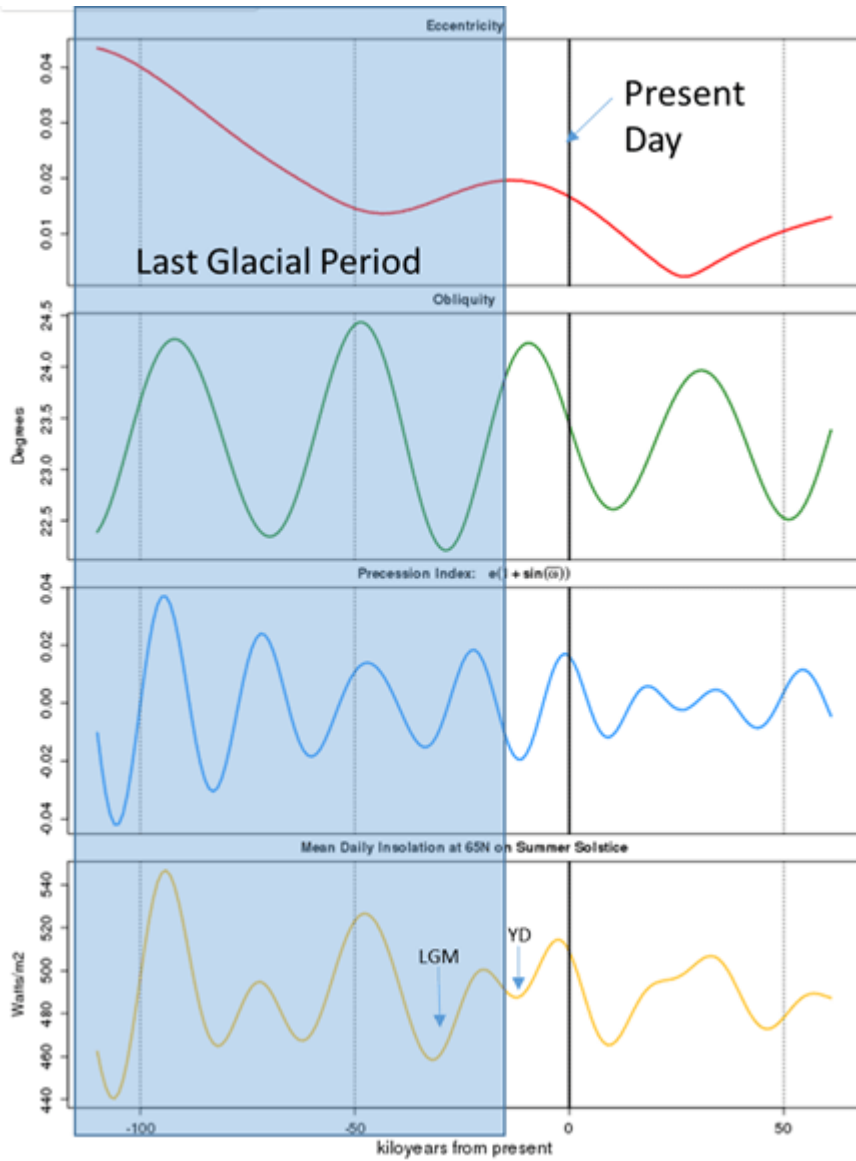
Figure 10: Orbital model versus CO2/CH4 based models; comparative performance over the last 10,000 years (from Ref.[19]).
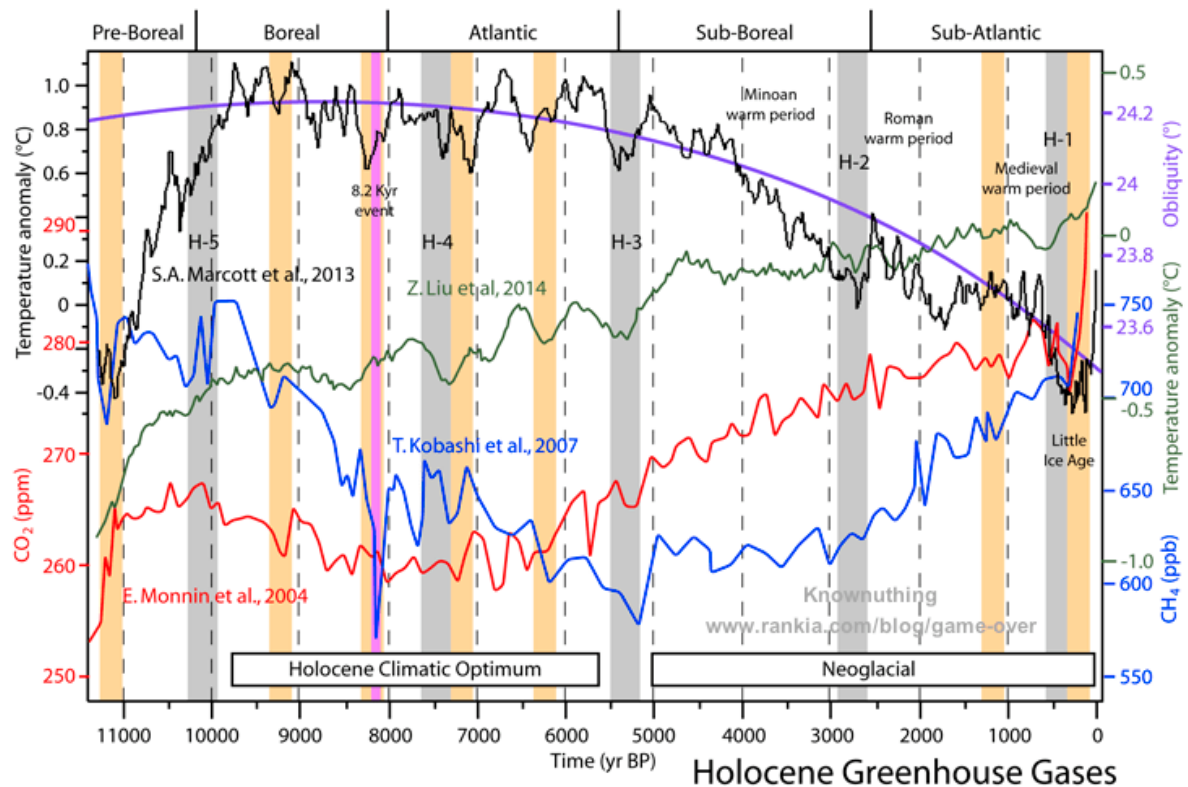
28. Summary

In summary, my predicted GAT increases due to a 50% increase in CO2 since pre-industrial (without feedback effects), though subject to a large error bar due to the crudity of my method, are nevertheless in broad agreement with the IPCC models - namely that about half of the observed 1.1°C GAT increase can be explained by the direct radiative forcing of CO2.

However, this corresponds to a reduction in infrared emissions into space of only ~1% at most (and perhaps far less), if this was not compensated by increases in temperature. The smallness of this percentage perturbation makes it difficult to rule out other sources of potential significance at this level of effect. This has been discussed above in terms of the extreme delicacy of the issue of changing cloud cover, and also the Sun's variations in radiance.

I make no claim regarding the actual cause of the observed 1.1°C increase in GAT. The above discussion serves only to indicate how small is the effect, and that, at such a small scale of perturbation, there are many contending contributary causes. In short, that the science cannot be regarded as settled.

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